On Solving Controlled-Invariance Problems in Dioids Using the PyMinMaxGD Python Scripts Library

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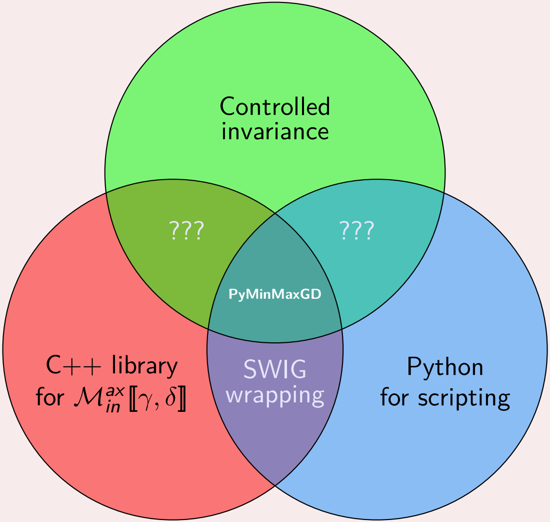
Agenda

Next Part

Weaving Theories and Technologies



Within the paradigm of discrete-event systems.

Why a New Toolbox?

Name	Implementation	Evolution from	Available dioids	Last version[10]
MinMaxGD (Hardouin, 2024)	C++		$\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$	03 Jul. 2024
ContainerMinMaxGD (Corronc, 2013)	C++	Fork of **MinMaxGD**	Intervals in $\overline{\mathbb{Z}}_{\min}$	21 Apr. 2011
MinMaxGDJS (Ferreira Cândido et al., 2017)	JavaScript wrapping over C++	Fork of **MinMaxGD**	$\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$	03 May 2019
ETVO (Cottenceau et al., 2022)	C++ / WebAssembly wrapping	Fork of **MinMaxGD**	$\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ and $\mathcal{ET}$ (see (Trunk, 2019, Chap. 5))	20 May 2023
Maxpluspy (Lahaye, 2019)	Python		$\overline{\mathbb{Z}}_{\max}$ and "$\overline{\mathbb{Z}}_{\max}$-automata"	27 Nov. 2019
PetriTUB (Bednar et al., 2024)	Python		$\overline{\mathbb{Z}}_{\max}$ and $\overline{\mathbb{Z}}_{\min}$, from Petri nets	14 Mar. 2024
MaxPlus Arithmetic Toolbox (Chancelier et al., 2015)	Already included in Scicoslab Toolbox	Fork of Scilab 4	$\overline{\mathbb{Z}}_{\max}$	03 Oct. 2015
MaxPlus.jl (Quadrat, 2024)	Julia	Port from **MaxPlus Arithmetic Toolbox**	$\overline{\mathbb{Z}}_{\max}$ and $\overline{\mathbb{Z}}_{\min}$	30 Apr. 2024
Max-Plus Algebra Toolbox (Stańczyk, 2016)	Matlab/Octave plug-in		$\overline{\mathbb{Z}}_{\max}$ and $\overline{\mathbb{Z}}_{\min}$	14 Jun. 2016
TropicalNumbers (Liu, 2023)	Julia		$\overline{\mathbb{Z}}_{\max}$, $\overline{\mathbb{Z}}_{\min}$, $(\mathbb{R}^+, max, \times)$ and $\mathcal{B}$	26 Sep. 2023

None of them[1] has all the following features:

1. handle both dating and counting dynamics;
2. open for extensions (plug-ins or open-source code);
3. scripting modelling for automation of benchmarks.

[10] As of 6 September 2024.

[1] Including other deprecated ones referenced in www.cmap.polytechnique.fr/~gaubert/MaxplusToolbox.html.

PyMinMaxGD : A C++ Wrapping in Python

SWIG Framework

Single interface file of about 900 lines of code, on top of the MinMaxGD library (Hardouin 2024) with roughly 5 000 lines of code (11 files - 4 classes and some high-level functions and constants).

Already Available Online

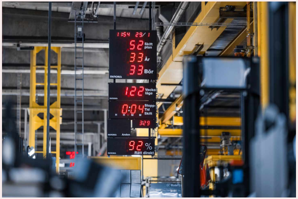
Git repository within a GitLab forge:

`https://gitlab.univ-nantes.fr/dioids/python-toolbox`

Anyone can join and contribute ! 😄

Next Part

Takt Time for Lean Management



- Takt time $\Rightarrow$ synchronise flow for client demand.

Why Working within a Dioid?

Two main phenomena:

- synchronization $\Rightarrow$ *max* function denoted $\oplus$ $(\varepsilon = -\infty)$;
- flow of time $\Rightarrow$ addition of numbers denoted $\otimes$ $(e = 0)$.

Dioid

Idempotent semi-ring[2].

Endowing $\oplus$ and $\otimes$ to a set of numerical values as its sum and product can make the modelling linear, *at the price of the inversion* of the first one.

Benefit

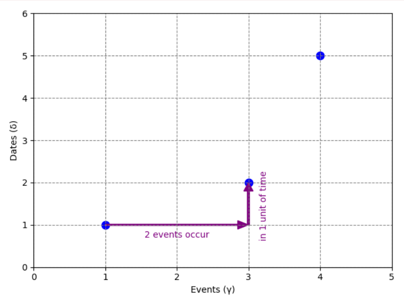
Nonlinear modelling (field)	$\Rightarrow$	linear modelling (dioid).
$max(a, b + c)$	$\Rightarrow$	$a \oplus (b \otimes c)$

[2] None of its two basic laws has to be invertible.

Formal Power Series in $\mathbb{B}[\![\gamma, \delta]\!]$

$\mathbb{B}[\![\gamma, \delta]\!]$ (Cohen et al. 1989)

Set of formal power series in two variables γ and δ with Boolean coefficients, of the form $\sum_{i=0}^{\infty} \gamma^i \delta^i$.



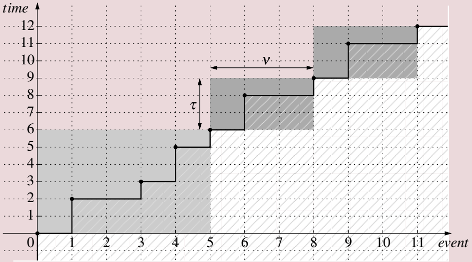
Graphical representation of $\gamma^1\delta^1 \oplus \gamma^3\delta^2 \oplus \gamma^4\delta^5 \in \mathbb{B}[\![\gamma, \delta]\!]$.

The time part of the modelling is now an ordinate.

$\Rightarrow$ Leads to compact modelling of information.

Semantics of Series in $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ in PM

Pseudo-Periodic Formal Power Series

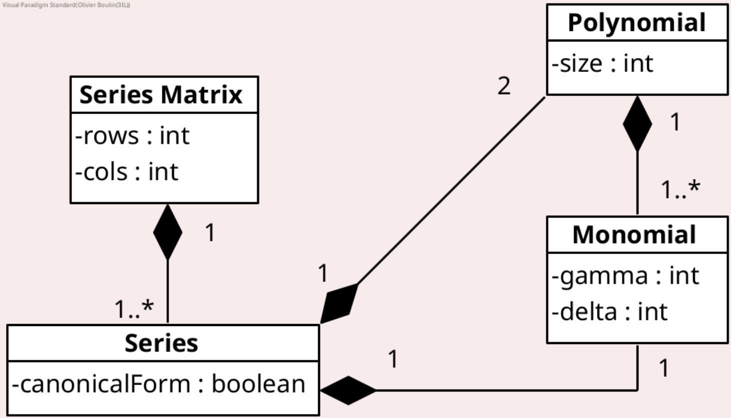


Graphical representation[3] of
$e \oplus \gamma\delta^2 \oplus \gamma^3\delta^3 \oplus \gamma^4\delta^5 \oplus (\gamma^5\delta^6 \oplus \gamma^6\delta^8)(\gamma^3\delta^3)^* \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$.

- Transient phase.
- Periodic behaviour of periodicity ν/τ. (here $\nu = 3$ and $\tau = 3$)

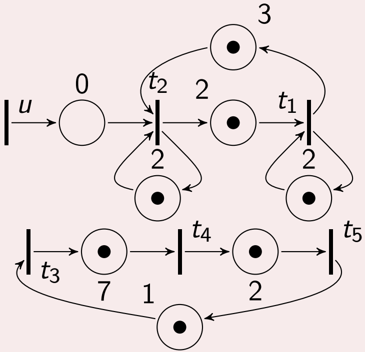
[3] e, within the context of $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ is equal to $\gamma^0\delta^0$, being the *epoch*.

Class Hierarchy



Timed Event Graphs

Timed Event Graphs (TEGs) define a class of timed Petri nets where there are no conflicts (Ramchandani 1974).



Next Part

Controlled Invariance In a Nutshell

Basically

The problem of synchronization is one of the many problems that can be expressed as a controlled-invariance problem.

Goal

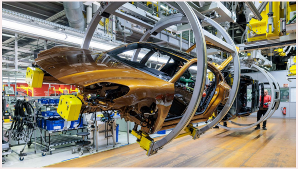
Find the greatest controlled-invariant module (we call it $\mathcal{V}^{\star}_{\mathcal{M}}(A, B)$, *a vector space in a dioid*).

- We can prove its existence.
- Not always calculable, i.e. in some cases, the $\mathcal{V}^{\star}_{\mathcal{M}}(A, B)$ is guaranteed to be finitely generated (Katz 2007).

For more theoretical details, see the content of the article and (Katz 2007), (Di Loreto et al. 2010), (Cárdenas, Loiseau, and C. Martinez 2017), (C. Martinez et al. 2022), and (Animobono et al. 2022).

Next Part

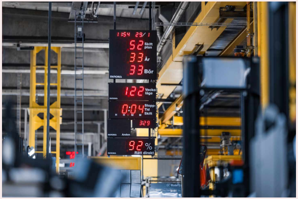
Synchronization within the Factory



The Plant

$$(\Sigma_1) \quad \begin{cases} x \Rightarrow \text{state} \\ y \Rightarrow \text{output} \end{cases}$$

Depending on an input u.

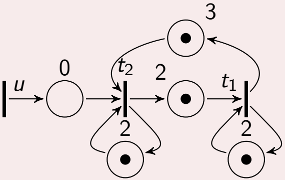


The Reference

$$(\Sigma_2) \quad \begin{cases} w \Rightarrow \text{state} \\ v \Rightarrow \text{output} \end{cases}$$

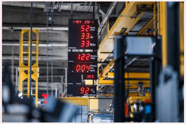
No dependence from u.

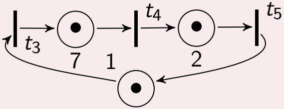
Modelling the States



The Plant ($\sum_1$)

$$x = A \otimes x \oplus B \otimes u$$

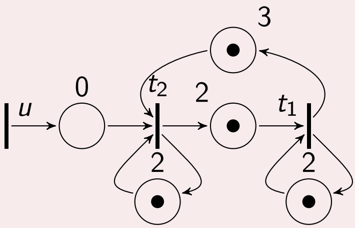




The Reference ($\sum_2$)

$$w = A_r \otimes w$$

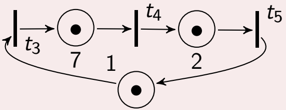
State Equation of the Plant (Σ_1)



For Equation $x = A \otimes x \oplus B \otimes u$.

$$A = \begin{pmatrix} \gamma^1\delta^2 & \gamma^1\delta^2 \\ \gamma^1\delta^3 & \gamma^1\delta^2 \end{pmatrix}, \; B = \begin{pmatrix} \varepsilon \\ e \end{pmatrix}$$

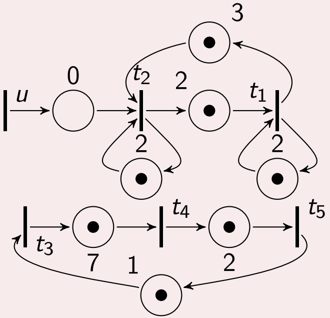
State Equation of the Reference (Σ_2)



For Equation $w = A_r \otimes w$

$$A_r = \begin{pmatrix} \varepsilon & \varepsilon & \gamma^1\delta^1 \\ \gamma^1\delta^7 & \varepsilon & \varepsilon \\ \varepsilon & \gamma^1\delta^2 & \varepsilon \end{pmatrix}$$

Extended System Σ



$$B_b = \begin{pmatrix} \varepsilon \\ e \\ \varepsilon \\ \varepsilon \\ \varepsilon \end{pmatrix}$$

$$A_b = \begin{pmatrix} \gamma^1\delta^2 & \gamma^1\delta^2 & \varepsilon & \varepsilon & \varepsilon \\ \gamma^1\delta^3 & \gamma^1\delta^2 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & \gamma^1\delta^1 \\ \varepsilon & \varepsilon & \gamma^1\delta^7 & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \gamma^1\delta^2 & \varepsilon \end{pmatrix}$$

Execution Logs

```
M =
  | 0               | 1               | 2               | 3               | 4               | 5               | 6               |
0 | ε_s             | (γ^0 δ^-5)[e]*  | ε_s             | (γ^0 δ^-2)[e]*  | ε_s             | (γ^0 δ^-5)[e]*  | (γ^0 δ^-4)[e]*  |
1 | (γ^0 δ^-2)[e]*  | ε_s             | (γ^0 δ^-4)[e]*  | ε_s             | (γ^0 δ^-5)[e]*  | ε_s             | ε_s             |
2 | (γ^0 δ^-2)[e]*  | (γ^0 δ^-5)[e]*  | (γ^0 δ^-4)[e]*  | (γ^0 δ^-2)[e]*  | (γ^0 δ^-5)[e]*  | (γ^0 δ^-5)[e]*  | (γ^0 δ^-4)[e]*  |
3 | (e)[e]*         | (e)[e]*         | (e)[e]*         | (e)[e]*         | (e)[e]*         | (e)[e]*         | (e)[e]*         |
4 | (γ^0 δ^-1)[e]*  | (γ^0 δ^-3)[e]*  | (e)[e]*         | (e)[e]*         | (γ^0 δ^-3)[e]*  | (γ^0 δ^-1)[e]*  | (e)[e]*         |
```

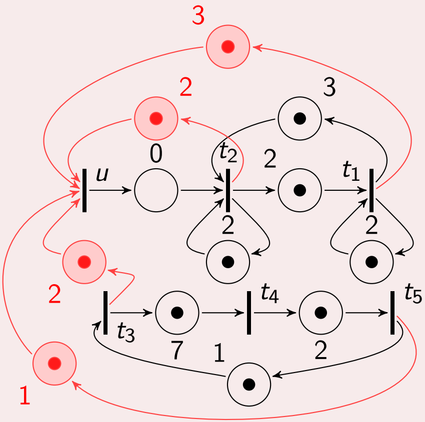
Execution Logs (Continued)

```
F=
  | 0              | 1              | 2              | 3              | 4              |
0 | (γ^1 δ^3)[e]*  | (γ^1 δ^2)[e]*  | (γ^1 δ^2)[e]*  | ε_s            | (γ^1 δ^1)[e]*  |
```

Corresponding Feedback Controller in $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$

$$F = \begin{pmatrix} \gamma^1\delta^3 & \gamma^1\delta^2 & \gamma^1\delta^2 & \varepsilon & \gamma^1\delta^1 \end{pmatrix}$$

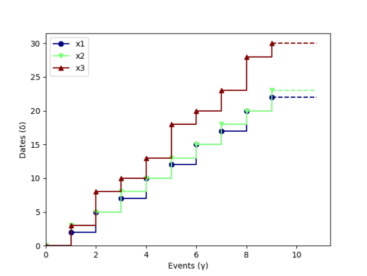
Whole System with the Feedback Controller



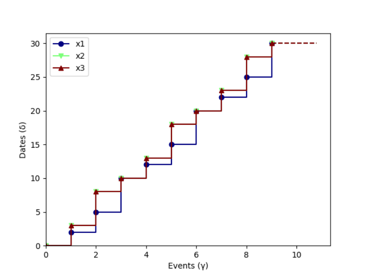
Reminder of the Feedback Controller

$$F = \begin{pmatrix} \gamma^1\delta^3 & \gamma^1\delta^2 & \gamma^1\delta^2 & \varepsilon & \gamma^1\delta^1 \end{pmatrix}$$

Simulations



Without synchronization.

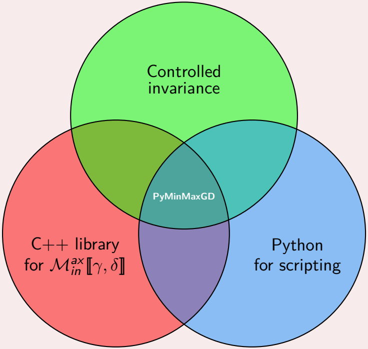


With synchronization ⇒ system has been slowed down.

- These pictures have been generated thanks to simple Python commands within our toolbox.

Next Part

Main Contribution and Future Work



Future Work

- Compatibility with all main OS's.
- Exploring other controlled-invariance problems:
  - disturbance rejection;
  - switching systems;
  - conditioned invariance problems (observability issues).

- Many thanks for your attention! Any question?

- More details in (Boutin, Claude Martinez, and Rakoto 2024), with fully reproducible examples thanks to the code available at[4]

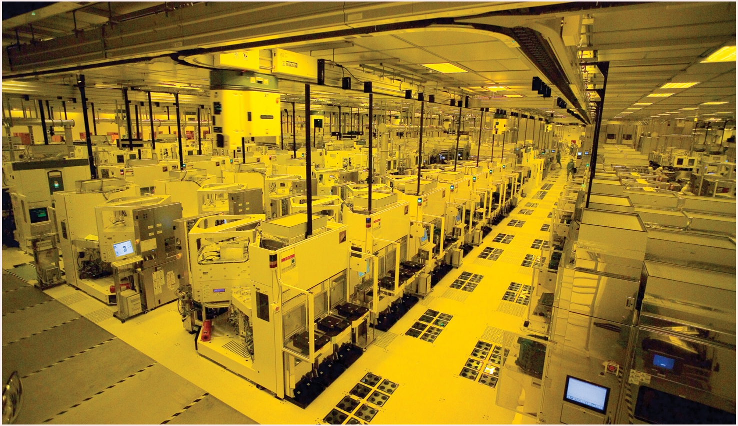
  `gitlab.univ-nantes.fr/dioids/python-toolbox`

[4] Work in progress. . .

Next Part

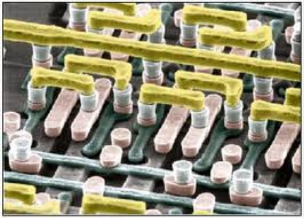
- **Extra content**
  - Other Kinds of Synchronization Problems
  - Formal Modelling
  - Technical Specifications
  - Historical Background
  - A few words on controlled invariance theory
  - Examples
- **Credits**

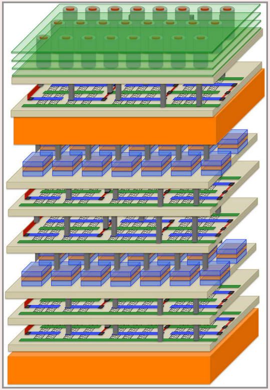
Production Based on Chemical Bath Deposition



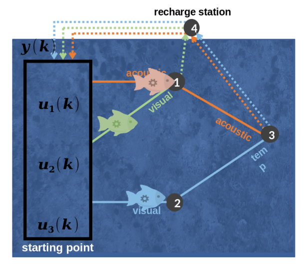
Strict time restrictions.

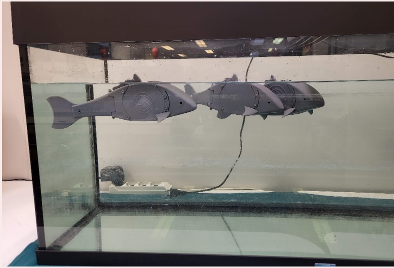
Focus on the "Wafers" and the Baths





Sinchronizing Members of a Schoal of Fishes





$\Rightarrow$ Exploration of confined submarine areas, using biomimetic marine vehicles. (Bartolucci et al. 2024)

Technical Specifications

GPLv3 license (Free Software Foundation, Inc. 2007). It has been compiled and installed with the following software requirements: `6.1 64-bit` (Super Long Time Support) Linux® kernel; `g++` 11.3.0; `SWIG` 4.0.2; `Python` 3.11.0; the `libpython3.11-dev` and `python-dev-is-python3` Linux packages and the `matplotlib`, `numpy`, `pillow` and `kiwisolver` Python libraries.

Historical Background of Invariance Control within Dioids

From Fields. . .

The concept of controlled invariance was presented first by Basile and Marro in (Basile and Marro 1969) and at the same time and independently by Wonham in (Wonham and Morse 1970) ((A, B)-invariance).

These works are the basis of the so called geometric approach, numerous classical control problems had original solutions in that framework (disturbance rejection, regulation...) (Wonham 1974; Basile and Marro 1992).

. . . to rings . . .

Many authors have extended the concept of controlled invariance to rings (Hautus 1982; Conte and Perdon 1995; Loreto, Lafay, and Loiseau 2008).

Historical Background of Invariance Control within Dioids

. . . to semi-rings

The first results using controlled invariance over a semi-ring, Dioids $\overline{\mathbb{Z}}_{max}$ and $\overline{\mathbb{N}}_{min}$, are due to Ricardo Katz, in (Gaubert and Katz 2004) and (Katz 2007).

In this study, we illustrate, for the first time, that controlled invariance can also be useful for the class of systems that are modelled with series in $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$.

Historical Background of Invariance Control within Dioids

Invariance and Feedback

In the case of a field, controlled invariance and static feedback (and linear) are equivalent (Wonham 1974; Basile and Marro 1992).

It is no longer the case for rings or semi-rings, only an implication remains, (Hautus 1982; Loreto, Lafay, and Loiseau 2008; Katz 2007).

Dynamic Feedback

Conte y Perdon introduced, in (Conte and Perdon 1995), the notion of dynamic feedback invariance that generalized the class of static control laws. Cardenas *et al.* (Cárdenas, Loiseau, and C. Martinez 2015) have extended this result for the $\mathbb{R}_{\max}$ dioid.

About sets

Image

A matrix $M \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n \times m}$ being given, we denote by $\mathrm{Im}\, M$ its image.

$$\mathrm{Im}\, M = \{x \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n} \mid \exists v \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{m},\ x = Mv\}\ .$$

The image by M of any set $\mathcal{S} \subset \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{m}$ is denoted $M\mathcal{S}$,
the preimage by M of any set $\mathcal{T} \subset \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{p}$ is denoted $M^{-1}\mathcal{T}$.

The difference of two sets $\mathcal{S}, \mathcal{S}' \subset \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{m}$, denoted $\mathcal{S} \ominus \mathcal{S}'$ is defined as $\{x" \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{m} \mid \exists x \in \mathcal{S}, x' \in \mathcal{S}', x" \oplus x' = x\}$.

$\mathcal{P}(\mathcal{S})$ is the collection of all subsets of set $\mathcal{S}$.

Modules in $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$

Module

A submodule of $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^n$ is said to be of finite type, or finitely generated, if there exists an integer q, and a matrix $M \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n \times q}$ so that $\mathcal{M} = \mathrm{Im}\, M$.

$$\mathrm{Cone}\,(C, D) = \{x \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^n \mid Cx = Dx\} \ ,$$

Theorem

Being given a module $\mathcal{M} \subset \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^n$, the following assertions are equivalent.

(i) There exists an integer q and a matrix $M \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n \times q}$ such that $\mathcal{M} = \mathrm{Im}\, M$.

(ii) There exists an integer p and matrices $C, D \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{p \times n}$ such that $\mathcal{M} = \mathrm{Cone}\,(C, D)$.

Controlled invariance

Controlled invariance

Consider a controlled system of the form

$$x = A \otimes x \oplus Bu \tag{1}$$

A set $\mathcal{M}$ is said to be controlled invariant if, for every vector $x \in \mathcal{M}$, there exists a control u such that the solution of system (1) is entirely in $\mathcal{M}$.

A set $\mathcal{S} \subset \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^n$ is controlled invariant if and only if $A\mathcal{S} \subset \mathcal{S} \ominus \mathrm{Im}\, B$
or equivalently, $\mathcal{S} \subset A^{-1}(\mathcal{S} \ominus \mathrm{Im}\, B)$.

Controlled invariance

Property

The union of any family of controlled invariant subsets in $\mathcal{P}(\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^n)$ is also a controlled invariant set.

As a consequence, every subset of $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^n$, say $\mathcal{S}$, contains a unique maximal controlled invariant subset, which contains every other controlled invariant subset of $\mathcal{S}$. This set is equal to the union of all the controlled invariant sets included in $\mathcal{S}$.

This maximal controlled invariant subset is denoted $\mathcal{V}_{\mathcal{S}}^{\star}(A,B)$.

Theorem, (Cárdenas, Loiseau, and C. Martinez 2017)

The following properties are verified.

(i) A module $\mathcal{M} \subset \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^n$ is controlled invariant if and only if: $$A\mathcal{M} \subset \mathcal{M} \ominus \operatorname{Im} B \,,$$

(ii) A module $\mathcal{M} \subset \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^n$ generated by the columns of matrix $M \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{n\times q}$ is controlled invariant if and only if there exist matrices $U \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{m\times q}$ and $V \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{q\times q}$ such that the following identity is verified: $$A \otimes M \oplus B \otimes U = M \otimes V \,.$$

(iii) A module $\mathcal{M} \subset \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^n$ such that $\mathcal{M} = \operatorname{Im} M = \operatorname{Cone}(R, Q)$, for matrices $M \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{n\times q}$ and $R, Q \in M \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{p\times n}$ is controlled invariant if and only if there exists matrix $U \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{m\times q}$ such that the following equality is verified: $$R(AM \oplus BU) = Q(AM \oplus BU) \,.$$

Controlled invariance

$\{\mathcal{V}_i\}$

$$\mathcal{V}_0 = \mathcal{S}\ ,\ \mathcal{V}_{i+1} = \mathcal{S} \cap A^{-1}(\mathcal{V}_i \ominus \mathrm{Im}\, B)\ ,\ \text{for } i \in \mathbb{N}\ . \qquad (2)$$

The sequence of subsets $\{\mathcal{V}_i\}$ is decreasing

- $\mathcal{V}_\omega := \bigcap_{i\in\mathbb{N}} \mathcal{V}_i$, satisfies $\mathcal{V}^\star_{\mathcal{S}}(A, B) \subset \mathcal{V}_\omega$;
- $\mathcal{V}^\star_{\mathcal{S}}(A, B) = \mathcal{V}_\omega$ if there exists an integer k such that $\mathcal{V}_k = \mathcal{V}_{k+1}$.

The successive terms of the sequence $\{\mathcal{V}_i\}$ are finitely generated modules.

Cárdenas, Loiseau, and C. Martinez 2017

We compute iteratively the matrices M_i and (R_i, Q_i) such that $\mathcal{V}_i = \mathrm{Im}\, M_i = \mathrm{Cone}(R_i, Q_i)$ seeking for a solution of the following equation:

$$\begin{pmatrix} R_{i-1}\,A & R_{i-1}\,B \\ R & \epsilon \end{pmatrix} \begin{pmatrix} M_i \\ U_i \end{pmatrix} = \begin{pmatrix} Q_{i-1}\,A & Q_{i-1}\,B \\ Q & \epsilon \end{pmatrix} \begin{pmatrix} M_i \\ U_i \end{pmatrix} . \qquad (3)$$

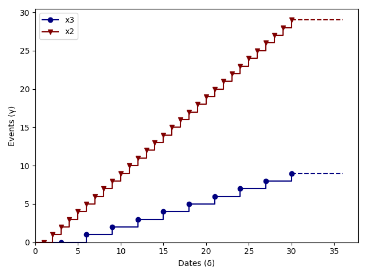
At each step, one computes M_i and U_i and then, one computes R_i et Q_i using theorem presented in page 38. The limit of the sequence is the intersection of modules $\mathcal{V}_i$.

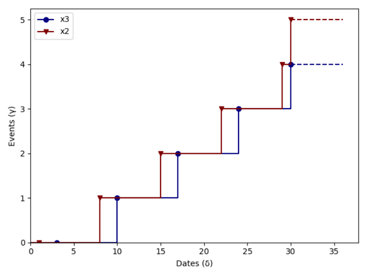
An example from Tebani *et al*, https://doi.org/10.1016/j.ejcon.2020.12.002

```
>>> exec(open("Tebani21EJC.py").read())
A=  | 0             | 1             | 2             | 3             | 4             | 5             | 6             |
0 | (γ^1 δ^1)[e]* | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           |
1 | (γ^0 δ^1)[e]* | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           |
2 | ε_s           | ε_s           | ε_s           | ε_s           | (γ^0 δ^1)[e]* | ε_s           | (γ^1 δ^1)[e]* |
3 | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | (γ^0 δ^1)[e]* |
4 | ε_s           | (γ^0 δ^1)[e]* | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           |
5 | ε_s           | ε_s           | (γ^0 δ^1)[e]* | ε_s           | ε_s           | ε_s           | ε_s           |
6 | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | (γ^0 δ^1)[e]* | ε_s           |
B=  | 0           |
0 | (e)[e]*       |
1 | ε_s           |
2 | ε_s           |
3 | ε_s           |
4 | ε_s           |
5 | ε_s           |
6 | ε_s           |
Gs=   | 0             | 1             | 2             | 3             | 4             | 5             | 6             |
0 | (e)[e]*       | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           |
1 | ε_s           | (e)[e]*       | (γ^1 δ^0)[e]* | ε_s           | ε_s           | ε_s           | ε_s           |
2 | ε_s           | ε_s           | (e)[e]*       | ε_s           | ε_s           | ε_s           | ε_s           |
3 | ε_s           | ε_s           | ε_s           | (e)[e]*       | ε_s           | ε_s           | ε_s           |
4 | ε_s           | ε_s           | ε_s           | ε_s           | (e)[e]*       | ε_s           | ε_s           |
5 | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | (e)[e]*       | ε_s           |
6 | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | (e)[e]*       |
```

```
  | 0             | 1             | 2             | 3             | 4             | 5             | 6             | 7             | 8             | 9             |
0 | (e)[e]*       | ε_s           | (e)[e]*       | ε_s           | (γ^1 δ^0)[e]* | ε_s           | (γ^1 δ^0)[e]* | (γ^2 δ^0)[e]* | (γ^1 δ^0)[e]* | ε_s           |
1 | ε_s           | ε_s           | (e)[e]*       | (e)[e]*       | ε_s           | ε_s           | ε_s           | ε_s           | (γ^1 δ^0)[e]* | (γ^1 δ^0)[e]* |
2 | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | (e)[e]*       | (e)[e]*       |
3 | ε_s           | (e)[e]*       | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           |
4 | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | (e)[e]*       | ε_s           | ε_s           | ε_s           |
5 | ε_s           | ε_s           | ε_s           | ε_s           | (e)[e]*       | (e)[e]*       | ε_s           | ε_s           | ε_s           | ε_s           |
6 | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | (e)[e]*       | ε_s           | ε_s           |
7
11
V=    | 0             | 1             | 2             | 3             | 4             | 5             | 6             | 7             | 8             | 9             |
0 | (γ^1 δ^1)[e]* | (γ^1 δ^1)[e]* | (γ^1 δ^1)[e]* | (γ^1 δ^1)[e]* | (γ^1 δ^1)[e]* | (γ^1 δ^1)[e]* | (γ^1 δ^1)[e]* | (γ^1 δ^1)[e]* | (γ^1 δ^1)[e]* | (γ^1 δ^1)[e]* |
Is R*A*M <= (Q*A*M)+ Q*B*V ? True
Is Q*A*M <= (R*A*M)+ R*B*V ? True
Is M A+B*V invariant ? True
Fc=
  | 0             | 1             | 2             | 3             | 4             | 5             | 6             |
0 | (γ^1 δ^1)[e]* | (γ^1 δ^1)[e]* | (γ^1 δ^1)[e]* | (γ^1 δ^1)[e]* | (γ^1 δ^1)[e]* | (γ^1 δ^1)[e]* | (γ^1 δ^1)[e]* |
```

Simulations





Without control.

With control ⇒ system has been slowed down.

- These pictures have been generated thanks to simple Python commands within our toolbox.

An example inspired from R. Katz and E. Wagneur

```
Ab =    | 0             | 1             | 2             | 3             | 4             | 5             |
0 | (γ^1 δ^0)[e]* | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           |
1 | ε_s           | (γ^1 δ^1)[e]* | ε_s           | ε_s           | ε_s           | ε_s           |
2 | ε_s           | ε_s           | (γ^1 δ^3)[e]* | ε_s           | ε_s           | ε_s           |
3 | ε_s           | ε_s           | ε_s           | (γ^1 δ^0)[e]* | ε_s           | ε_s           |
4 | ε_s           | ε_s           | ε_s           | ε_s           | (γ^1 δ^1)[e]* | ε_s           |
5 | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | (γ^1 δ^2)[e]* |
S =    | 0             | 1             | 2             | 3             | 4             | 5             |
0 | (e)[e]*       | ε_s           | ε_s           | (e)[e]*       | ε_s           | ε_s           |
1 | ε_s           | (e)[e]*       | ε_s           | ε_s           | (e)[e]*       | ε_s           |
2 | ε_s           | ε_s           | (e)[e]*       | ε_s           | ε_s           | (e)[e]*       |
T =    | 0             | 1             | 2             | 3             | 4             | 5             |
0 | ε_s           | ε_s           | ε_s           | (e)[e]*       | ε_s           | ε_s           |
1 | ε_s           | ε_s           | ε_s           | ε_s           | (e)[e]*       | ε_s           |
2 | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | (e)[e]*       |
S0 =    | 0             | 1             | 2             | 3             | 4             | 5             |
0 | ε_s           | ε_s           | ε_s           | (e)[e]*       | ε_s           | ε_s           |
1 | ε_s           | ε_s           | ε_s           | ε_s           | (e)[e]*       | ε_s           |
2 | ε_s           | ε_s           | ε_s           | ε_s           | ε_s           | (e)[e]*       |
3 | (e)[e]*       | ε_s           | ε_s           | (e)[e]*       | ε_s           | ε_s           |
4 | ε_s           | (e)[e]*       | ε_s           | ε_s           | (e)[e]*       | ε_s           |
5 | ε_s           | ε_s           | (e)[e]*       | ε_s           | ε_s           | (e)[e]*       |
```

```
We compute the solution of H3 x = Q3 x...
Is M3 controlled invariant ? False
M 4 :
   | 0               | 1               | 2               | 3               | 4               | 5               | 6                |
0  | ε_s             | ε_s             | ε_s             | (γ^4 δ^0)[e]*   | ε_s             | ε_s             | ε_s              |
1  | ε_s             | ε_s             | ε_s             | ε_s             | (γ^4 δ^4)[e]*   | ε_s             | ε_s              |
2  | ε_s             | ε_s             | ε_s             | ε_s             | ε_s             | (γ^4 δ^8)[e]*   | ε_s              |
3  | (e)[e]*         | ε_s             | ε_s             | (γ^4 δ^0)[e]*   | ε_s             | ε_s             | (γ^11 δ^12)[e]*  |
4  | ε_s             | (e)[e]*         | ε_s             | ε_s             | (γ^4 δ^4)[e]*   | ε_s             | (γ^11 δ^11)[e]*  |
5  | ε_s             | ε_s             | (e)[e]*         | ε_s             | ε_s             | (γ^4 δ^12)[e]*  | (γ^11 δ^13)[e]*  |
cac
Size of H4: 24 x 7
Size of Q4: 24 x 7
We compute the solution of H4 x = Q4 x...
Is M4 controlled invariant ? False
M 5 :
   | 0               | 1               | 2               | 3               | 4               | 5               | 6                |
0  | ε_s             | ε_s             | ε_s             | (γ^5 δ^0)[e]*   | ε_s             | ε_s             | ε_s              |
1  | ε_s             | ε_s             | ε_s             | ε_s             | (γ^5 δ^5)[e]*   | ε_s             | ε_s              |
2  | ε_s             | ε_s             | ε_s             | ε_s             | ε_s             | (γ^5 δ^10)[e]*  | ε_s              |
3  | (e)[e]*         | ε_s             | ε_s             | (γ^5 δ^0)[e]*   | ε_s             | ε_s             | (γ^14 δ^15)[e]*  |
4  | ε_s             | (e)[e]*         | ε_s             | ε_s             | (γ^5 δ^5)[e]*   | ε_s             | (γ^14 δ^14)[e]*  |
5  | ε_s             | ε_s             | (e)[e]*         | ε_s             | ε_s             | (γ^5 δ^15)[e]*  | (γ^14 δ^17)[e]*  |
cac
Size of H5: 24 x 7
Size of Q5: 24 x 7
We compute the solution of H5 x = Q5 x...
Is M5 controlled invariant ? False
M 6 :
   | 0               | 1               | 2               | 3               | 4               | 5               | 6                |
0  | ε_s             | ε_s             | ε_s             | (γ^6 δ^0)[e]*   | ε_s             | ε_s             | ε_s              |
1  | ε_s             | ε_s             | ε_s             | ε_s             | (γ^6 δ^6)[e]*   | ε_s             | ε_s              |
2  | ε_s             | ε_s             | ε_s             | ε_s             | ε_s             | (γ^6 δ^12)[e]*  | ε_s              |
3  | (e)[e]*         | ε_s             | ε_s             | (γ^6 δ^0)[e]*   | ε_s             | ε_s             | (γ^17 δ^18)[e]*  |
4  | ε_s             | (e)[e]*         | ε_s             | ε_s             | (γ^6 δ^6)[e]*   | ε_s             | (γ^17 δ^17)[e]*  |
5  | ε_s             | ε_s             | (e)[e]*         | ε_s             | ε_s             | (γ^6 δ^18)[e]*  | (γ^17 δ^21)[e]*  |
cac
Size of H6: 24 x 7
Size of Q6: 24 x 7
We compute the solution of H6 x = Q6 x...
Is M6 controlled invariant ? False
M 7 :
   | 0               | 1               | 2               | 3               | 4               | 5               | 6                |
0  | ε_s             | ε_s             | ε_s             | (γ^7 δ^0)[e]*   | ε_s             | ε_s             | ε_s              |
1  | ε_s             | ε_s             | ε_s             | ε_s             | (γ^7 δ^7)[e]*   | ε_s             | ε_s              |
2  | ε_s             | ε_s             | ε_s             | ε_s             | ε_s             | (γ^7 δ^14)[e]*  | ε_s              |
3  | (e)[e]*         | ε_s             | ε_s             | (γ^7 δ^0)[e]*   | ε_s             | ε_s             | (γ^20 δ^21)[e]*  |
4  | ε_s             | (e)[e]*         | ε_s             | ε_s             | (γ^7 δ^7)[e]*   | ε_s             | (γ^20 δ^20)[e]*  |
5  | ε_s             | ε_s             | (e)[e]*         | ε_s             | ε_s             | (γ^7 δ^21)[e]*  | (γ^20 δ^25)[e]*  |
cac
```

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